STAD DEPARTMENT PH.D. QUALIFYING EXAMINATION – THEORETICAL STATISTICS January 2025

Instructions

1. There are two parts to this exam: STA 584 and STA 684. You should attempt to answer all questions on the blank paper provided.

2. Write only on one side of the answer paper. Begin each problem on a new sheet and indicate the problem number and subpart (a,b,c..). Please show all of your work and state any assumptions, if necessary.

3. If you redo a problem, please cross out any answers you do not want graded. Any crossed out work will be ignored.

4. If a particular theorem is required for proving something, please state the theorem and justify why it is applicable.

5. Any new notations must be defined before starting the problem.

6. When you have completed the exam, please collate all pages according to the problem numbers. You should then number the pages separately. You should also hand in this exam with your answers.

Name _____

STA-584 Section

Problem 1 (3 points x 4 = 12 points).

a. An office worker types three letters and then the three corresponding envelopes. In a hurry, the worker places one letter at random into each envelope. What is the probability that at least one letter is in the correct envelope?

b. A chemist wishes to detect an impurity in a certain compound that she is making. There is a test that detects an impurity with probability 0.9; however, this test indicates that an impurity is there when it is not about 5% of the time. The chemist produces compounds with the impurity about 20% of the time. A compound is selected at random from the chemist's output. The test indicates that the impurity is present. What is the probability that the compound actually has the impurity?

c. A warship has 5 guided missiles. A single guided missile will hit its target 80% of the time. Assume it takes three successful hits to destroy the target. What is the probability that the target is destroyed with the 4th or 5th missile?

d. Phone calls to 911 follow a Poisson Distribution at a rate of 2 calls every 5 minutes. In one hour, what is the probability of exactly 24 calls?

Problem 2 (3 points x 3 = 9 points).

Let $f(x,y) = 21x^2y^3, \;\; 0 < x < y < 1, \;\;\; 0$ elsewhere, be the joint pdf of X and Y.

a. Determine if X and Y are independent. Prove your conclusion.

b. Determine the condition distribution of $f(X|Y)$ including the support.

c. Determine the conditional mean of X given Y. $E(X|Y)$ including the support.

Problem 3 (3 points x 3 = 9 points).

Let X be a continuous random variable with pdf $f(x) = 3x^2$ for $0 < x < 1$.

a. What is the probability density and support of $Y = X^2$?

b. What is the Expected Value of $Y = X^2$?

c. What is the Median of $Y = X^2$?

4. (4+3+3 = 10 points)

Let X_1 and X_2 have the joint pdf $f(x_1, x_2) = 2$ with $0 < x_1 < x_2 < 1$

a. Find the joint pdf of $U = \frac{X_1+X_2}{2}$ and $V = X_2$.

b. Find the pdf of U and show this is a legitimate pdf with the correct support.

c. Determine the expected value of U.