PH.D. QUALIFYING EXAMINATION – THEORETICAL STATISTICS
Time: 9:00 AM – 1:00 PM, Thursday, August 22, 2024

Instructions

- There are two parts in this exam: STA 584 and STA 684. You are to answer all questions. The raw score for each part will be converted to its percentage.
- Write on one side only. Begin each subpart on a new sheet with the problem number clearly labeled. You must show all your work and justifications correctly and completely to receive full credit. Partial credits may be given for partially correct solutions.
- For each problem/subproblem, hand in only the answer you want to be graded. Crossed-out work will be ignored. Failure to follow this instruction for a problem will result in a zero score for the problem.
- If a theorem is applied, you must state it, identify its assumption(s) and conclusion(s), and justify why it is applicable. New notations must be defined before use. You need to specify the support of the distribution in your answer.
- When finished, please collate all pages based on problem labels and then number the pages accordingly. Hand in also the exam paper.

By signing below, I hereby acknowledge that I have completely read and fully understand the instructions.

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Signature

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Printed Name

Part A - STA 584

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1. (8 *points*) Let X and Y be independent and identically distributed uniform random variables on the interval (0,1). Define $U = X + Y$ and $W = Y$.

1.a) Find the joint pdf of U and W. (3 *points*)

1.b) Find the pdf of U. (3 *points*)

1.c) Are U and W independent? Justify your answer. (2 *points*)

2. (12 *points*, 2 *each* unless otherwise stated) If the joint pdf of X and Y is given by

$$f(x, y) = ky \text{ for } 0 < y < x < 2, \text{ zero elsewhere.}$$

2.a) Find the constant k.

2.b) Find the marginal pdf of Y. Specify its support and check whether your answer can serve as a pdf before proceeding.

2.c) Find conditional pdf of X given $Y = y$, where $0 < y < 2$. (1 *point)*

2.d) Derive the conditional mean of X given $Y = y$.

2.e) Derive the conditional variance of X given $Y = y$. (3 *points*)

2.f) Find the expected value $E\left(\frac{1}{Y}\right)$.

3. (8 *points*) Let the joint pdf of X and Y be given by

$$f(x, y) = x + y \text{ for } 0 < x < 1 \text{ and } 0 < y < 1, \quad \text{zero elsewhere.}$$

3.a) Assuming that $E(Y) = 7/12$, find $\text{Cov}(X, Y)$, the covariance between X and Y. (4 points)

3.b) Find $\text{Var}(X)$, the variance of X. (2 points)

3.c) Define $W = -\ln(X)$. Find the cumulative distribution function of W. (2 points)

4. (6 *points,* 3 *points each)* Solve the following problems.

4.a) Derive the moment generating function of a gamma distribution with mean $\alpha\beta$ and variance $\alpha\beta^2$.

4.b) If $\mathrm{Var}(X) = 4$, $\mathrm{Var}(Y) = 2$, $\mathrm{Var}(Z) = 3$, $\mathrm{Cov}((X, Y) = 1$, $\mathrm{Cov}(X, Z) = -2$, and Y and Z are independent. Define $W = X - 2Y$ and $U = -2X + Y - Z$. Find the covariance between W and U.

5. (6 *points, 2 points each*) Solve all the following problems.

5.a) Forty percent of those who have studied earn an A, and only 5% of those who have not studied earn an A. It is known that 80% of students study for a test. What is the probability that a student who earns an A actually did not study for the test?

5.b) As part of an air-pollution survey, an inspector decides to examine the exhaust of 6 of a company's 12 trucks. If 3 of the company's trucks emit excessive amounts of pollutants, what is the probability that x of them will be included in the inspector's sample?

5.c) The probability that a person living in a city owns a dog is 0.4. Find the probability that the sixth person randomly interviewed in the city is the third one to own a dog. What is the name of the probability distribution you've employed?

6. (8 *points*) Suppose X and Y have the following joint probability distribution.

		X	
	$f(x, y)$	2	4
Y	0	0.10	0.15
	1	0.20	0.30
	2	0.10	0.15

6.a) Find is the cumulative distribution function of Y. (3 points)

6.b) Find the conditional variance of X given $Y = 1$. (4 points)

6.c) Find the probability distribution of $X + Y$. ((1 point)